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1) $T \propto \dfrac{1}{|\vec{x}|} \Rightarrow T(x,y,z) = \dfrac{k}{\sqrt{x^2+y^2+z^2}}$

and $T(1,2,2) = 120$

$\Rightarrow \dfrac{k}{\sqrt{1^2+2^2+2^2}} = 120 \Rightarrow \dfrac{k}{3} = 120 \Rightarrow k = 360.$

a) Now we want to find the directional derivative of T at $(1,2,2)$ in the direction of

$\vec{v} = \langle 2,1,3 \rangle - \langle 1,2,2 \rangle$

$= \langle 1,-1,1 \rangle$

Then the unit vector in the direction of $\vec{v}$ is given by $\vec{u} = \dfrac{\langle 1,-1,1 \rangle}{\sqrt{3}}$

$$\nabla T(x,y,z) = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k} = \frac{-360x}{(x^2+y^2+z)^{-3/2}}\hat{i} + \frac{-360y}{(x^2+y^2+z)^{-3/2}}\hat{j} + \frac{-360z}{(x^2+y^2+z)^{-3/2}}\hat{k}$$

Then, $\nabla T(1,2,2) = -\dfrac{40}{3}\langle 1,2,2 \rangle$

Finally, $D_{\vec{u}}T(1,2,2) = \nabla T(1,2,2) \cdot \vec{u} = -\dfrac{40}{3}\langle 1,2,2 \rangle \cdot \dfrac{1}{\sqrt{3}}\langle 1,-1,1 \rangle = \dfrac{-40}{3\sqrt{3}}$

b) From part a, $\nabla T = \dfrac{-360x}{(x^2+y^2+z)^{-3/2}}\hat{i} + \dfrac{-360y}{(x^2+y^2+z)^{-3/2}}\hat{j} + \dfrac{-360z}{(x^2+y^2+z)^{-3/2}}\hat{k}$

$$= \frac{-360}{(x^2+y^2+z)^{-3/2}}\langle x,y,z \rangle$$

and as $\langle x,y,z \rangle$ is the position vector of point (x,y,z),
the vector $-\langle x,y,z \rangle$ and thus ∇T always points towards the origin.

2) Let $F(x,y,z) = x^2 - 2y^2 + z^2 + yz$.

Then, $x^2 - 2y^2 + z^2 + yz = 2$ is a level surface of F and

$\nabla F(x,y,z) = \langle 2x, -4y+z, 2z+y \rangle$

a) $\nabla F(2,1,-1) = \langle 4,-5,-1 \rangle$ is a normal vector for the tangent plane at $(2,1,-1)$, so equation of tangent plane is given by

$\nabla F(2,1,-1) . \langle x-2, y-1, z+1\rangle = 0$

$\Rightarrow \langle 4,-5,-1\rangle . \langle x-2, y-1, z+1\rangle = 0$

$\Rightarrow 4(x-2) -5(y-1) - 1(z+1) = 0 \Rightarrow 4x-5y-z = 4.$

b) The normal line has direction $\langle 4,-5,-1\rangle$, so the equation of line is

$\vec{r}(t) = \langle 2,1,-1\rangle + t\langle 4,-5,-1\rangle$

$= \langle 2+4t, 1-5t, -1-t\rangle$

and the symmetric equations are given by $\frac{x-2}{4} = \frac{y-1}{-5} = \frac{z+1}{-1}$

3) a) $f(x,y) = (1+xy)(x+y) = x+y+x^2y+xy^2$

• $f_x = 1+2xy+y^2$, $f_y = 1+x^2+2xy$

$f_{xx} = 2y$, $f_{xy} = 2x+2y$, $f_{yy} = 2x$.

Critical points $f_x = 0$ and $f_y = 0 \Rightarrow 1+2xy+y^2 = 0 \longrightarrow ①$ & $1+x^2+2xy = 0 \longrightarrow ②$

Substracting ② from ① gives $y^2-x^2 = 0 \Rightarrow y = \pm x$.

When $y = x$, $1+2xy+y^2 = 0 \Rightarrow 1+2x^2+x^2 = 0 \Rightarrow 1+3x^2 = 0$ which has no real soln.

When $y = -x$, $1+2xy+y^2 = 0 \Rightarrow 1-x^2 = 0 \Rightarrow x = 1,-1$.

So critical points are $(1,-1)$ and $(-1,1)$.

• $D(1,-1) = f_{xx}(1,-1). f_{yy}(1,-1) - \left[f_{xy}(1,-1)\right]^2 = (-2)(2) - 0 < 0$

• $D(-1,1) = 2(-2) - 0 < 0$

So, $(-1,1)$ and $(1,-1)$ are saddle points.

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b) $f(x,y) = x\sin y$

$f_x = \sin y$, $f_y = x\cos y$
$f_{xx} = 0$, $f_{xy} = \cos y$, $f_{yy} = -x\sin y$.

<u>Critical points</u> $f_x = 0$ and $f_y = 0$ $\Rightarrow$ $\sin y = 0$ & $x\cos y = 0$
$\Rightarrow$ $y = n\pi$ and $x = 0$, where n is an integer.

- So critical points are $(0, n\pi)$, n an integer.

$D(0,n\pi) = -\cos^2(n\pi) < 0$, so each critical points is a saddle point.

4) a) $f(x,y) = 2x^3 + y^4$ is <u>continuous</u> on the <u>closed</u> disc $D = \{(x,y) \mid x^2+y^2 \leq 1\}$, abs max & min exist.

- $f_x(x,y) = 6x^2$ and $f_y(x,y) = 4y^3$. So, $f_x = 0$, $f_y = 0$ $\Rightarrow$ $x = 0, y = 0$

Therefore, the only critical point in the disc is $(0,0)$ and $f(0,0) = 0$.

On the boundary, $x^2+y^2 = 1$ $\Rightarrow y^2 = (1-x^2)$,
$g(x) = f(x,y) = 2x^3 + (1-x^2)^2 = x^4 + 2x^3 - 2x^2 + 1$, $-1 \leq x \leq 1$

- $g'(x) = 4x^3 + 6x^2 - 4x = 0$ $\Rightarrow x = 0, -2$ or $\frac{1}{2}$

- $g(0) = f(0, \pm 1) = 1$
- $g\left(\frac{1}{2}\right) = f\left(\frac{1}{2}, \pm\frac{\sqrt{3}}{2}\right) = \frac{13}{16}$

- $g(-1) = f(-1,0) = -2$ abs min
- $g(1) = f(1,0) = 2$ abs max

b) Since $f(x,y) = 1 + 4x - 5y$ is continuous on closed D, abs max & min exist.

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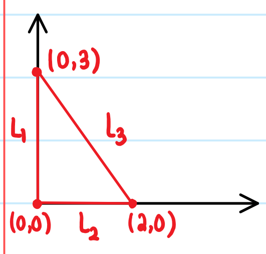
• $f_x = 4$, $f_y = -5$, so there are no critical points inside D.



$\underline{L_1}$ $x=0$, $g(y) = f(0,y) = 1-5y$, $0 \le y \le 3$ and since it is a line extreme values occur at endpoints.

• $g(0) = f(0,0) = 1$

• $g(3) = f(0,3) = -14$

$\underline{L_2}$ $y=0$ and $h(x) = f(x,0) = 1+4x$, $0 \le x \le 2$

• $h(0) = f(0,0) = 1$ • $h(2) = f(2,0) = 9$

$\underline{L_3}$ $y = 3 - \frac{3}{2}x$ and $p(x) = f\left(x, 3-\frac{3}{2}x\right) = \frac{23}{2}x - 14$, $0 \le x \le 2$

• $p(0) = f(0,3) = -14$ • $p(2) = f(2,0) = 9$

Thus abs max is $f(2,0) = 9$ & abs min is $f(0,3) = -14$.

5) We want to maximize $V(x,y) = 4xy\sqrt{36-9x^2-9y^2}$ w/ (x,y,z) in the first quadrant.

Then,

$$V_x = 4\left[(36-9x^2-36y^2)^{1/2} + -9x^2y(36-9x^2-36y^2)^{-1/2}\right] = 4 \cdot \frac{36y - 18x^2y - 36y^3}{(36-9x^2-36y^2)^{1/2}}$$

$$V_y = 4 \cdot \frac{(36x - 9x^3 - 72xy^2)}{(36-9x^2-36y^2)^{1/2}}$$

. Setting $V_x = 0$ gives $y=0$ or $y^2 = \frac{2-x^2}{2}$ but $y>0$, so $y \neq 0$.

Then substituting y into $V_y = 0$ gives $x^2 = \frac{4}{3} \Rightarrow x = \pm\frac{2}{\sqrt{3}}$ but since $x>0$, $x = \frac{2}{\sqrt{3}}$, $y = \frac{1}{\sqrt{3}}$

Then $z^2 = \frac{(36-12-12)}{4} \Rightarrow \underline{z = \sqrt{3}}$

The fact that this gives the maximum value can either be determined by 2^{nd} derivative test (which probably will be horrible computation) or as follows:

$V(x,y)$ is continuous on a closed, bounded region so abs max occurs on the boundary or at critical point. On the boundary $f(x,y) = 0$ and at the critical point $V(x,y) = 4 \cdot \frac{1}{\sqrt{3}} \cdot \frac{2}{\sqrt{3}} \cdot \sqrt{12} = \frac{16}{\sqrt{3}}$ which is the maximum volume.

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6) a) $f(x,y,z) = xyz$; $g(x,y,z) = x^2 + 2y^2 + 3z^2 = 6$

$\nabla f = \langle yz, xz, xy \rangle$, $\lambda \nabla g = \langle 2\lambda x, 4\lambda y, 6\lambda z \rangle$

Then,

$f = \nabla g \Rightarrow yz = 2\lambda x$, $xz = 4\lambda y$, $xy = 6\lambda z$. $\Rightarrow \lambda = \frac{yz}{2x} = \frac{xz}{4y} = \frac{xy}{6z}$

$\Rightarrow x^2 = 2y^2$ and $z^2 = \frac{2}{3}y^2$

Thus, $x^2 + 2y^2 + 3z^2 = 6 \Rightarrow 2y^2 + 2y^2 + 3 \cdot \left(\frac{2}{3}y^2\right) = 6 \Rightarrow 6y^2 = 6 \Rightarrow y = \pm 1$

$\Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$ and $z^2 = \frac{2}{3} \Rightarrow z = \pm\sqrt{\frac{2}{3}}$

Thus the possible points are $\left(-2,-1,-\sqrt{\frac{2}{3}}\right)$, $\left(-2,-1,\sqrt{\frac{2}{3}}\right)$, $\left(2,-1,-\sqrt{\frac{2}{3}}\right)$

$\left(2,1,-\sqrt{\frac{2}{3}}\right)$ $\left(2,-1,\sqrt{\frac{2}{3}}\right)$ $\left(-2,1,-\sqrt{\frac{2}{3}}\right)$ $\left(-2,1,\sqrt{\frac{2}{3}}\right)$ $\left(2,1,\sqrt{\frac{2}{3}}\right)$

Then maximum value of f is $\frac{2}{\sqrt{3}}$ occuring when all x,y,z are positive or exactly two are negative.

Then minimum value of f is $-\frac{2}{\sqrt{3}}$ occuring when all x,y,z are negative or exactly one is negative.

b) $f(x,y,z) = yz + xy$

$g(x,y,z) = xy = 1$

$h(x,y,z) = y^2 + z^2 = 1$

$\nabla f = \langle y, z+x, y \rangle$

$\nabla g = \langle y, x, 0 \rangle$, $\nabla h = \langle 0, 2y, 2z \rangle$

$\nabla f = \lambda \nabla g + \mu \nabla h \Rightarrow \langle y, x+z, y \rangle = \langle \lambda y, \lambda x, 0 \rangle + \langle 0, 2\mu y, 2\mu z \rangle$

$\Rightarrow \langle y, x+z, y \rangle = \langle \lambda y, \lambda x + 2\mu y, 2\mu z \rangle$

- $y = \lambda y \Rightarrow \lambda y - y = 0 \Rightarrow y(\lambda - 1) = 0 \Rightarrow y = 0, \lambda = 1.$
  But $y \neq 0$ since $xy = 1$.

- $x + z = \lambda x + 2\mu y \Rightarrow z = 2\mu y \Rightarrow \mu = \frac{z}{2y}$
- $y = 2\mu z \Rightarrow \mu = \frac{y}{2z}$

$\Rightarrow \frac{y}{2z} = \frac{z}{2y} \Rightarrow y^2 = z^2$

Since $h(x,y,z) = y^2 + z^2 = 1 \Rightarrow 2y^2 = \pm 1 \Rightarrow y = \pm\frac{1}{\sqrt{2}}$ and $z = \pm\frac{1}{\sqrt{2}}$

Then $g(x,y,z) = 1 \Rightarrow xy = 1 \Rightarrow x = \pm\sqrt{2}$

The possible points are $\left(\pm\sqrt{2}, \pm\frac{1}{\sqrt{2}}, +\frac{1}{\sqrt{2}}\right), \left(\pm\sqrt{2}, \pm\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

Then the maximum of f subject to the constraint is $f\left(\pm\sqrt{2}, \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right) = \frac{3}{2}$

and minimum is $f\left(\pm\sqrt{2}, \pm\frac{1}{\sqrt{2}}, \mp\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$.

7) The distance from the origin to a point (x,y,z) is $d(x,y,z) = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$
Instead of minimizing or maximizing d, we can work w/ $d^2 = f(x,y,z) = x^2 + y^2 + z^2$
(Convince yourself that min (or max) of d occurs at the same point as min (or max) of d^2).

Since the point lies on both the plane and the paraboloid, the function f is subject to two constraints $g(x,y,z) = x + y + 2z = 2$ & $h(x,y,z) = x^2 + y^2 - z = 0$

Then, $\nabla f = \lambda \nabla g + \mu \nabla h \Rightarrow$

$\Rightarrow \langle 2x, 2y, 2z \rangle = \langle \lambda, \lambda, 2\lambda \rangle + \langle 2\mu x, 2\mu y, -\mu \rangle$

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So, $2x = \lambda + 2\mu x$ (1), $2y = \lambda + 2\mu y$ (2), $2z = 2\lambda - \mu$ (3), $x+y+2z = 2$ (4), $x^2+y^2-z=0$ (5)

(1) − (2) $\Rightarrow$ $x-y = \mu(x-y)$ $\Rightarrow$ $x-y=0$ or $\mu = 1$. Suppose $x-y \neq 0$, then $\mu = 1$.
Then plugging $\mu = 1$ into (3) we get $2z = 2\lambda - 1$ $\Rightarrow$ $\lambda = z + \frac{1}{2}$ $\rightarrow$ (6)
Plugging $\mu = 1$ into (1) we get $\lambda = 1$ $\Rightarrow$ Then from 6 we get $z = -\frac{1}{2}$
If we plug in $z = -\frac{1}{2}$ into (5) we obtain $x^2+y^2+\frac{1}{2} = 0$ which
has no real solution. Therefore our initial assumption $x-y \neq 0$ has no solution and so we have
$x-y=0$ or $x=y$.

Then (4) becomes $2x+2z=2$ $\Rightarrow$ $x+z=1$ $\Rightarrow$ $z = 1-x$ $\rightarrow$ (7)
and (5) becomes $2x^2 - z = 0$ $\Rightarrow$ $z = 2x^2$ $\rightarrow$ (8)

From (7) and (8) we get $2x^2 = 1-x$ $\Rightarrow$ $2x^2+x-1=0$ $\Rightarrow$ $(2x-1)(x+1)=0$ $\Rightarrow$ $x = \frac{1}{2}$, $x=-1$

When $x = \frac{1}{2}$, $y = \frac{1}{2}$ and $z = 1 - \frac{1}{2} = \frac{1}{2}$

When $x=-1$, $y=-1$ and $z = 1-(-1) = 2$
(We are not interested in the exact values of λ, μ)
• So the two points we need to check are $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ and $(-1,-1,2)$.

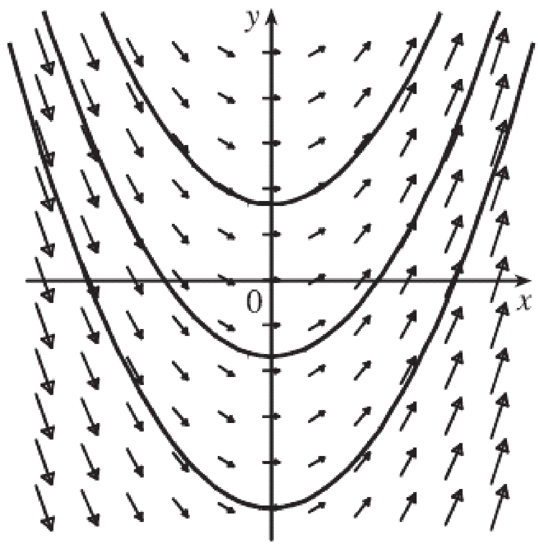
• $f(1/2, 1/2, 1/2) = 3/4$
$f(-1,-1,2) = 6$

$(1/2, 1/2, 1/2)$ is the pt on the ellipse nearest to $(0,0,0)$ &
$(-1,-1,2)$ is the pt on the ellipse farthest from $(0,0,0)$.

8) For this question, the question on the handout is slightly different than the one in the book.

Book

a) To the right, you can see the sketch of the vector field $\vec{F}(x,y) = \hat{i} + x\hat{j}$ along w/ approximate flow lines which appear to be parabolas.



b) If $x = x(t)$ and $y = y(t)$ are parametric equation of the flow lines, then the velocity vector of the flow line at (x,y) is $\langle x'(t), y'(t) \rangle$.

Since these velocity vectors are the same as our vector field, we have

$$x'(t)\,\hat{i} + y'(t)\,\hat{j} = \hat{i} + x\hat{j} \Rightarrow \frac{dx}{dt} = 1 \text{ and } \frac{dy}{dt} = x$$

Then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = x$

c) $\frac{dy}{dx} = x \Rightarrow dy = x\,dx \Rightarrow \int dy = \int x\,dx \Rightarrow y = \frac{1}{2}x^2 + C$

Since, the particle starts at $(0,0)$, we get

$0 = \frac{1}{2}(0)^2 + C \Rightarrow C = 0$, so the path the particle follows is $y = \frac{1}{2}x^2$.

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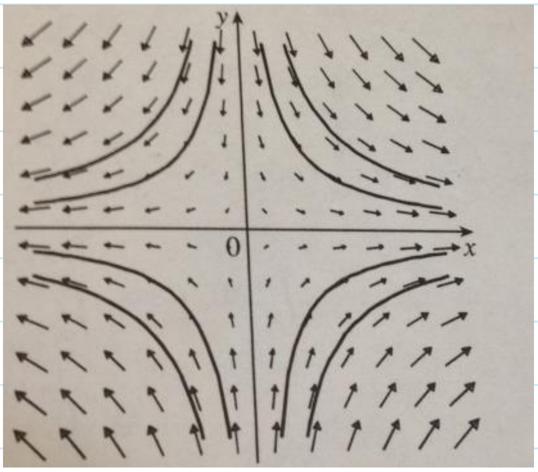
a) To the right, you can see the sketch of the vector field $\vec{F}(x,y) = x\hat{i} - y\hat{j}$ along w/ approximate flow lines which appear to be hyperbolas with shape similar to the graph of $y = \pm\frac{1}{x}$.



b) If $x = x(t)$ and $y = y(t)$ are parametric equation of the flow lines, then the velocity vector of the flow line at (x,y) is $\langle x'(t), y'(t)\rangle$.

Since these velocity vectors are the same as our vector field, we have

$$x'(t)\,\hat{i} + y'(t)\,\hat{j} = x\hat{i} - y\hat{j} \quad \Rightarrow \quad \frac{dx}{dt} = x \quad \text{and} \quad \frac{dy}{dt} = -y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x} \quad \Rightarrow \quad \frac{dy}{y} = \frac{-dx}{x} \quad \Rightarrow \quad \int \frac{dy}{y} = \int -\frac{dx}{x}$$

$$\Rightarrow \ln|y| = -\ln|x| + C \quad \Rightarrow \ln|y| + \ln|x| = C \quad \Rightarrow \ln|xy| = C \quad \Rightarrow xy = e^C = K$$

So the equation of the flow line is $xy = K$ and if the flow lines passes through $(1,1)$, then

$$1.1 = K \Rightarrow K = 1. \quad \Rightarrow xy = 1 \quad \Rightarrow y = \frac{1}{x}, \; x > 0.$$

9a) C is a semicircle centered at $(0,0)$ w/ radius 4, so we can parametrize C as $x = 4\cos t$, $y = 4\sin t$, $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$.

$$\text{Then, } \int_C xy^4\,ds = \int_{-\pi/2}^{\pi/2} (4\cos t)(4\sin t)^4 \sqrt{(-4\sin t)^2 + (4\cos t)^2}\,dt = \int_{-\pi/2}^{\pi/2} 4^5 \cos t \sin^4 t \sqrt{16(\sin^2 t + \cos^2 t)}\,dt$$

$$= 4^6 \int_{-\pi/2}^{\pi/2} \sin^4 t \cos t \; dt \quad . \quad u = \sin t, \; du = \cos t \, dt$$

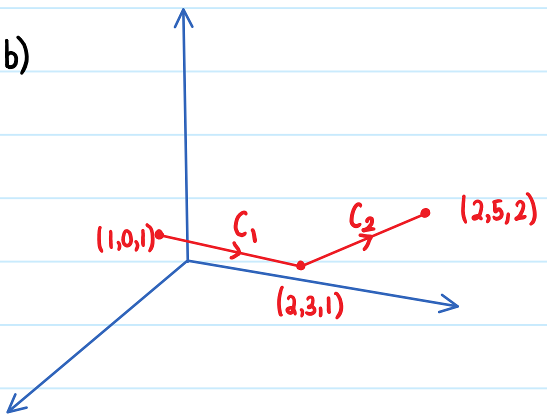
$$t = -\pi/2, \; u = -1$$

$$t = \pi/2, \; u = 1$$

$$= 4^6 \int_{-1}^{1} u^4 du$$

$$= 4^6 \cdot \frac{u^5}{5} \Big]_{-1}^{1} = \frac{2 . 4^6}{5}$$

b)



• $\underline{C_1}$ $\vec{r}(t) = (1-t)\langle 1,0,1\rangle + t\langle 2,3,1\rangle$

$$= \langle 1-t+2t, 3t, 1-t+t \rangle$$

$$= \langle 1+t, 3t, 1 \rangle \quad , \; 0 \le t \le 1$$

$$x = 1+t, \; y = 3t, \; z = 1$$

$$dx = dt, \; dy = 3dt, \; dz = 0$$

• $\underline{C_2}$ $x = 2, \; y = 3+2t, \; z = 1+t \Rightarrow dx = 0, \; dy = 2dt, \; dz = dt$

Then, $\int_C (x+yz)\,dx + 2x\,dy + xyz\,dz = \int_{C_1} (x+yz)\,dx + 2x\,dy + xyz\,dz + \int_{C_2} (x+yz)\,dx + 2x\,dy + xyz\,dz$

$$= \int_0^1 (1+t+3t.(1))\,dt + 2(1+t).3dt + \int_0^1 2.(2).2dt + (2)(3+2t)(1+t)\,dt$$

$$= \int_0^1 10t+7 \; dt + \int_0^1 4t^2+10t+14 \; dt = \int_0^1 4t^2+20t+21 \; dt = \left[\frac{4t^3}{3} + 10t^2 + 21t\right]_0^1 = \frac{97}{3}$$

10) We can use the parametrization $x = 2\cos t, \; y = 2\sin t, \; -\frac{\pi}{2} \le t \le \frac{\pi}{2}$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \sqrt{4(\sin^2 t + \cos^2 t)}\;dt = 2\,dt$$

Then, $m = \int_C k\,ds = \int_{-\pi/2}^{\pi/2} 2k\,dt = 2k(\pi)$.

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$$M_y = \int_C x\,k\,ds = \int_{-\pi/2}^{\pi/2} 2\sin t \cdot 2k\,dt = 4k\left[\sin t\right]_{-\pi/2}^{\pi/2} = 8k$$

$$M_x = \int_C ky\,ds = \int_{-\pi/2}^{\pi/2} k\,2\sin t \cdot 2\,dt = 4k\int_{-\pi/2}^{\pi/2} \sin t\,dt = 0 \quad \left(\begin{array}{l}\text{odd function}\\ \text{over symm interval}\end{array}\right)$$

$$\bullet\ (\bar{x},\bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right) = \left(\frac{8k}{2\pi k}, 0\right) = \left(\frac{4}{\pi}, 0\right)$$

11) $\vec{r}(t) = (1-t)\vec{r}_0 + t\vec{r}_1 \quad , 0 \le t \le 1$

$= (1-t)\langle 1,0,0\rangle + t\langle 3,4,2\rangle$

$= \langle 1+2t, 4t, 2t\rangle, \ 0 \le t \le 1$

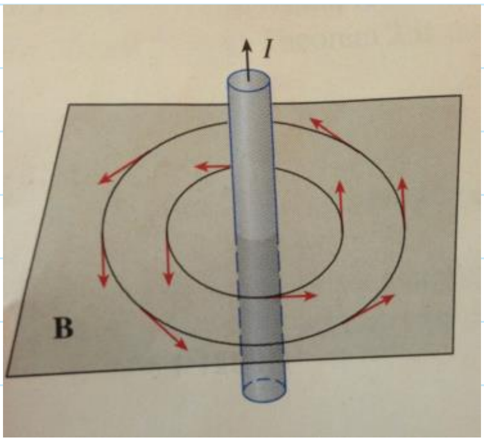
$\vec{r}'(t) = \langle 2,4,2\rangle$

$$W = \int_C \vec{F}\cdot d\vec{r} = \int_C \vec{F}(\vec{r}(t))\cdot\vec{r}'(t)\,dt = \int_0^1 \langle 6t, 1+4t, 1+6t\rangle\cdot\langle 2,4,2\rangle\,dt$$

$$= \int_0^1 (12t + 4(1+4t) + 2(1+6t))\,dt$$

$$= \int_0^1 (40t+6)\,dt = 26.$$

12) We will use the orientation given in the figure to the right.



Note that $\vec{B}$ is tangent to any circle that lies in the plane perpendicular to the wire,

$\vec{B} = |\vec{B}|\vec{T}$, where $\vec{T}$ is the unit tangent vector to the circle C.

Since C can be parametrize by $x = r\cos\theta$; $y = r\sin\theta$, $0 \le \theta \le 2\pi$ and therefore $\vec{T} = \dfrac{\vec{r}'(t)}{|\vec{r}'(t)|} = \langle -\sin\theta, \cos\theta\rangle$.

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Then,

$$\int_C \vec{B}.d\vec{r} = \int_0^{2\pi} |\vec{B}|.\langle -\sin\theta, \cos\theta \rangle . \langle -r\sin\theta, r\cos\theta \rangle \, d\theta$$

$$= \int_0^{2\pi} |B| . r(\sin^2\theta + \cos^2\theta) \, d\theta = \int_0^{2\pi} |\vec{B}|.r \, d\theta = 2\pi r |\vec{B}|$$

Since, $\int_C \vec{B}.d\vec{r} = \mu_0 I \Rightarrow 2\pi r |\vec{B}| = \mu_0 I \Rightarrow |\vec{B}| = \frac{\mu_0 I}{2\pi r}$.